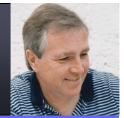


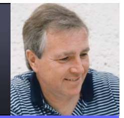
Grundrechenarten 1

$$(4-3\cdot i)+(2\cdot i-8)=(4+(-8))+((-3)+2)i$$

$$=(-4)+(-1)i=-4-i$$

$$\frac{2-3i}{4-i}=\frac{(2-3i)(4+i)}{(4-i)(4+i)}=\frac{(8+3)+(2-12)i}{17}$$

$$=\frac{11}{17}-\frac{10}{17}i$$



Grundrechenarten 2

$$(4+i)(3+2\cdot i)(1-i) = (10+11i)(1-i) = 21+i$$

$$3(-1+4\cdot i)-2(7-i) = (-3+12\cdot i)-(14+2i)$$

$$= -17+14\cdot i$$

$$(i-2)(2(1+i)-3(i-1)) = (i-2)((2+2i)-(3i-3))$$

$$= (i-2)(5-i) = -9+7i$$

Grundrechenarten 3

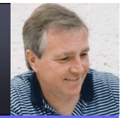
$$\left(\frac{1+i}{1-i}\right)^2 - 2\left(\frac{1-i}{1+i}\right)^3 = \left(\frac{2i}{2}\right)^2 - 2\left(\frac{-2i}{2}\right)^3 = (i)^2 - 2(-i)^3$$

$$= -1-2i$$

Hinweis: $i^3 = i^2 \cdot i = -i \qquad (-i)^3 = -i^3 = i$

$$\frac{(2+i)(3-2i)(1+2i)}{(1-i)^2} = \frac{(8-i)(1+2i)}{(-2i)} = \frac{10+15i}{-2i} = \frac{(10+15i)i}{(-2i)i}$$

$$= \frac{(10i-15)}{2} = -\frac{15}{2}+5i$$



Wurzeln

$$\sqrt{i} = \alpha + \beta i \quad \Big|^2$$

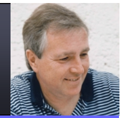
$$i = (\alpha^2 - \beta^2) + 2\alpha\beta\, i \quad \Rightarrow \left| \begin{array}{c} \alpha^2 - \beta^2 = 0 \\ 2\alpha\beta = 1 \end{array} \right| \quad \Rightarrow \left| \begin{array}{c} \alpha^2 - \beta^2 = 0 \\ \beta = \dfrac{1}{2\alpha} \end{array} \right|$$

$$\Rightarrow \alpha^2 - \left(\frac{1}{2\alpha} \right)^2 = 0$$

$$\Rightarrow 4\alpha^4 - 1 = 0 \quad \Rightarrow \alpha^4 = \frac{1}{4}$$

$$\Rightarrow \alpha^2 = \frac{1}{2} \quad \Rightarrow \alpha = \pm\frac{\sqrt{2}}{2} \quad \Rightarrow \beta = \pm\frac{\sqrt{2}}{2}$$

$$\sqrt{i} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \quad \vee \quad \sqrt{i} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$$



Quadratische Gleichungen

$$z^2 + (i-2)z + (3-i) = 0$$

$$x^2 + px + q = 0$$

$$x_{1,2} = -\frac{p}{2} \pm \frac{\sqrt{p^2 - 4q}}{2}$$

$$z_{1,2} = -\frac{i-2}{2} \pm \frac{\sqrt{(i-2)^2 - 4(3-i)}}{2}$$

$$= (1 - \frac{1}{2}i) \pm \frac{\sqrt{(3-4i) - (12-4i)}}{2}$$

$$= (1 - \frac{1}{2}i) \pm \frac{\sqrt{(-9)}}{2}$$

$$= (1 - \frac{1}{2}i) \pm \frac{3i}{2}$$

$$z_1 = (1-i) \vee z_2 = (1-2i)$$

Konjugieren 1

Seien $z_1 = a + bi \qquad z_2 = c + di$

Zeige: $(z_1 \cdot z_2)^* = z_1^* \cdot z_2^*$

$$(z_1 \cdot z_2)^* = ((a+bi)(c+di))^*$$

$$= ((ac-bd)+(ad+bc)i)^*$$

$$= (ac-bd)-(ad+bc)i$$

$$z_1^* \cdot z_2^* = (a-bi)(c-di)$$

$$= (ac-bd)-(ad+bc)i$$

Konjugieren 2

Seien $z_1 = a + bi \qquad z_2 = c + di$

Zeige: $\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$

$$\left(\frac{z_1}{z_2}\right)^* = \left(\frac{(ac+bd)+(-ad+bc)i}{c^2+d^2}\right)^*$$

$$= \left(\frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i\right)^*$$

$$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}i$$

$$\frac{z_1^*}{z_2^*} = \frac{a-bi}{c-di}$$

$$= \frac{(a-bi)(c+di)}{c^2+d^2}$$

$$= \frac{ac+bd}{c^2+d^2} + \frac{ad-bc}{c^2+d^2}i$$

$$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}i$$