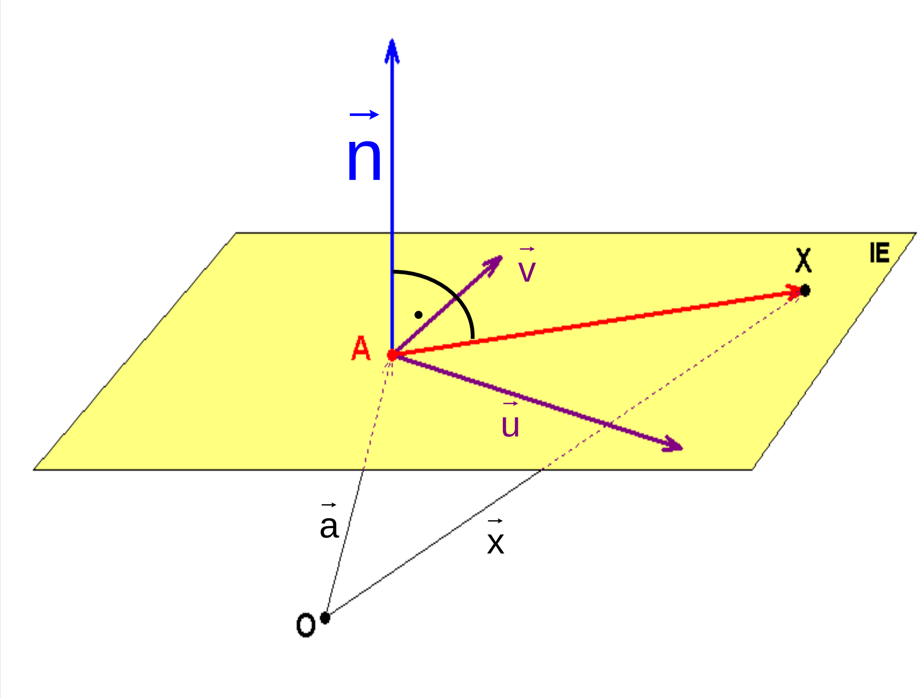
Ebenengleichungen:



Parameterform:

$$E: \quad \vec{x} = \vec{a} + \lambda\vec{u} + \mu\vec{v} \qquad ; \quad \lambda, \mu \in \mathbb{R}$$

Normalenform:

$$E: \quad \vec{n} * (\vec{x} - \vec{a}) = 0$$

Koordinatenform:

$$E: \quad n_1 \cdot x_1 + n_2 \cdot x_2 + n_3 \cdot x_3 + d = 0$$

$$n_1, n_2, n_3 \in \mathbb{R}$$

Hesse-Normalenform:

$$E: \quad \overrightarrow{n_0} * (\vec{x} - \vec{a}) = 0$$

$$\text{mit } \left|\overrightarrow{n_0}\right| = 1 \text{ und } \overrightarrow{n_0} * \vec{a} \geq 0$$

Zusätzlich gilt:

$$\vec{n} \perp \vec{u} \qquad \vec{n} \perp \vec{v}$$

Parameterform → Normalenform → Koordinatenform → Hesse-Nf

Beispiel:

$$E: \quad \vec{x} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -5 \\ 8 \end{pmatrix} \quad ; \ \lambda, \mu \in \mathbb{R}$$

$$\vec{n} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 2 \\ 0 & -5 & 8 \end{vmatrix} = \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} \qquad E: \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}\right) = 0$$

$$E: \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \begin{pmatrix} x_1 - 5 \\ x_2 - 2 \\ x_3 - 3 \end{pmatrix} = 0 \qquad E: \quad 10 \cdot x_1 - 8 \cdot x_2 - 5 \cdot x_3 - 19 = 0$$

Parameterform → Normalenform → Koordinatenform → Hesse-Nf

Beispiel:

$$E: \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}\right) = 0 \qquad E: \; 10 \cdot x_1 - 8 \cdot x_2 - 5 \cdot x_3 - 19 = 0$$

$$\vec{n} = \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} \quad \left|\vec{n}\right| = \sqrt{189} \quad \vec{n_0} = \frac{1}{\sqrt{189}} \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} \quad \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} = 50 - 16 - 15 \geq 0$$

$$E: \frac{1}{\sqrt{189}} \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}\right) = 0 \qquad E: \frac{10 \cdot x_1 - 8 \cdot x_2 - 5 \cdot x_3 - 19}{\sqrt{189}} = 0$$

Koordinatengleichung→Normalenform→Parameterform

$$E:\quad 10\cdot x_1 - 8\cdot x_2 - 5\cdot x_3 - 19 = 0$$

$$\vec{n} = \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} \quad A(1{,}9\,|\,0\,|\,0) \in E \quad \Rightarrow \vec{a} = \begin{pmatrix} 1{,}9 \\ 0 \\ 0 \end{pmatrix} \qquad E:\ \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 1{,}9 \\ 0 \\ 0 \end{pmatrix}\right) = 0$$

$$A(1{,}9\,|\,0\,|\,0) \in E \qquad B(1\,|\,-1\,|\,-0{,}2) \in E \qquad C(2\,|\,0\,|\,0{,}2) \in E$$

$$\Rightarrow \vec{u} = \overrightarrow{AB} = \begin{pmatrix} -0{,}9 \\ -1 \\ -0{,}2 \end{pmatrix} \qquad \vec{v} = \overrightarrow{AC} = \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix}$$

$$E:\quad \vec{x} = \begin{pmatrix} 1{,}9 \\ 0 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix} + \varepsilon \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix} \qquad \nu, \varepsilon \in \mathbb{R}$$

Äquivalenz von Ebenengleichungen in Parameterform

$$E_1: \quad \vec{x} = \begin{pmatrix} 1{,}9 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix} \qquad \lambda, \mu \in \mathbb{R}$$

$$E_2: \quad \vec{x} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} + \nu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 \\ -5 \\ 8 \end{pmatrix} \qquad ; \quad \nu, \varepsilon \in \mathbb{R}$$

zu zeigen:

$$(5|2|3) \in E_1 \qquad \begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix}; \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \text{ sind l.a.} \qquad \begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix}; \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix} \begin{pmatrix} 0 \\ -5 \\ 8 \end{pmatrix} \text{ sind l.a.}$$

Äquivalenz von Ebenengleichungen in Parameterform

$$E_1: \quad \vec{x} = \begin{pmatrix} 1{,}9 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix} \qquad \lambda, \mu \in \mathbb{R}$$

$$(5 \mid 2 \mid 3) \in E_1 \quad : \quad \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1{,}9 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 0{,}9 & 1 & 3{,}1 \\ 1 & 1 & 2 \\ 0{,}2 & 0{,}4 & 3 \end{array}\right) \qquad \left(\begin{array}{cc|c} -0{,}1 & 0 & 1{,}1 \\ 1 & 1 & 2 \\ 0{,}2 & 0{,}4 & 3 \end{array}\right) \quad \begin{array}{l} \Rightarrow 0{,}1\lambda = -1{,}1 \;\Rightarrow\; \lambda = -11 \\ \Rightarrow \mu = 13 \\ 0{,}2 \cdot (-11) + 0{,}4 \cdot 13 = 3 \\ \qquad\qquad\qquad 3 = 3 \end{array}$$

Äquivalenz von Ebenengleichungen in Parameterform

$$E_1:\quad \vec{x}=\begin{pmatrix}1,9\\0\\0\end{pmatrix}+\lambda\begin{pmatrix}0,9\\1\\0,2\end{pmatrix}+\mu\begin{pmatrix}1\\1\\0,4\end{pmatrix}\qquad \lambda,\mu\in\mathbb{R}$$

$$E_2:\quad \vec{x}=\begin{pmatrix}5\\2\\3\end{pmatrix}+\nu\begin{pmatrix}1\\0\\2\end{pmatrix}+\varepsilon\begin{pmatrix}0\\-5\\8\end{pmatrix}\quad ;\ \nu,\varepsilon\in\mathbb{R}$$

$$\begin{pmatrix}0,9\\1\\0,2\end{pmatrix};\begin{pmatrix}1\\1\\0,4\end{pmatrix}\begin{pmatrix}1\\0\\2\end{pmatrix}\text{ sind l.a.}$$

$$\begin{vmatrix}0,9 & 1 & 1\\1 & 1 & 0\\0,2 & 0,4 & 2\end{vmatrix}\begin{matrix}0,9 & 1\\1 & 1\\0,2 & 0,4\end{matrix}=1,8+0+0,4-(0,2+0+2)=0$$

Äquivalenz von Ebenengleichungen in Parameterform

$$E_1: \vec{x} = \begin{pmatrix} 1{,}9 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix} \quad \lambda, \mu \in \mathbb{R}$$

$$E_2: \vec{x} = \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} + \nu \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \varepsilon \begin{pmatrix} 0 \\ -5 \\ 8 \end{pmatrix} \quad ; \ \nu, \varepsilon \in \mathbb{R}$$

$$\begin{pmatrix} 0{,}9 \\ 1 \\ 0{,}2 \end{pmatrix}; \begin{pmatrix} 1 \\ 1 \\ 0{,}4 \end{pmatrix} \begin{pmatrix} 0 \\ -5 \\ 8 \end{pmatrix} \text{ sind l.a.}$$

$$\left| \begin{matrix} 0{,}9 & 1 & 0 \\ 1 & 1 & -5 \\ 0{,}2 & 0{,}4 & 8 \end{matrix} \right| \begin{matrix} 0{,}9 & 1 \\ 1 & 1 \\ 0{,}2 & 0{,}4 \end{matrix} = 7{,}2 - 1 + 0 - (0 - 1{,}8 + 8) = 0$$

Äquivalenz von Ebenengleichungen in Normalenform

$$E_1: \quad \overrightarrow{n_1} * (\vec{x} - \vec{a}) = 0 \qquad\qquad E_2: \quad \overrightarrow{n_2} * (\vec{x} - \vec{b}) = 0$$

$$\boxed{E_1 = E_2 \Leftrightarrow \overrightarrow{n_1} = k \cdot \overrightarrow{n_2} \quad \wedge\ A \in E_2 \text{ bzw. } B \in E_1}$$

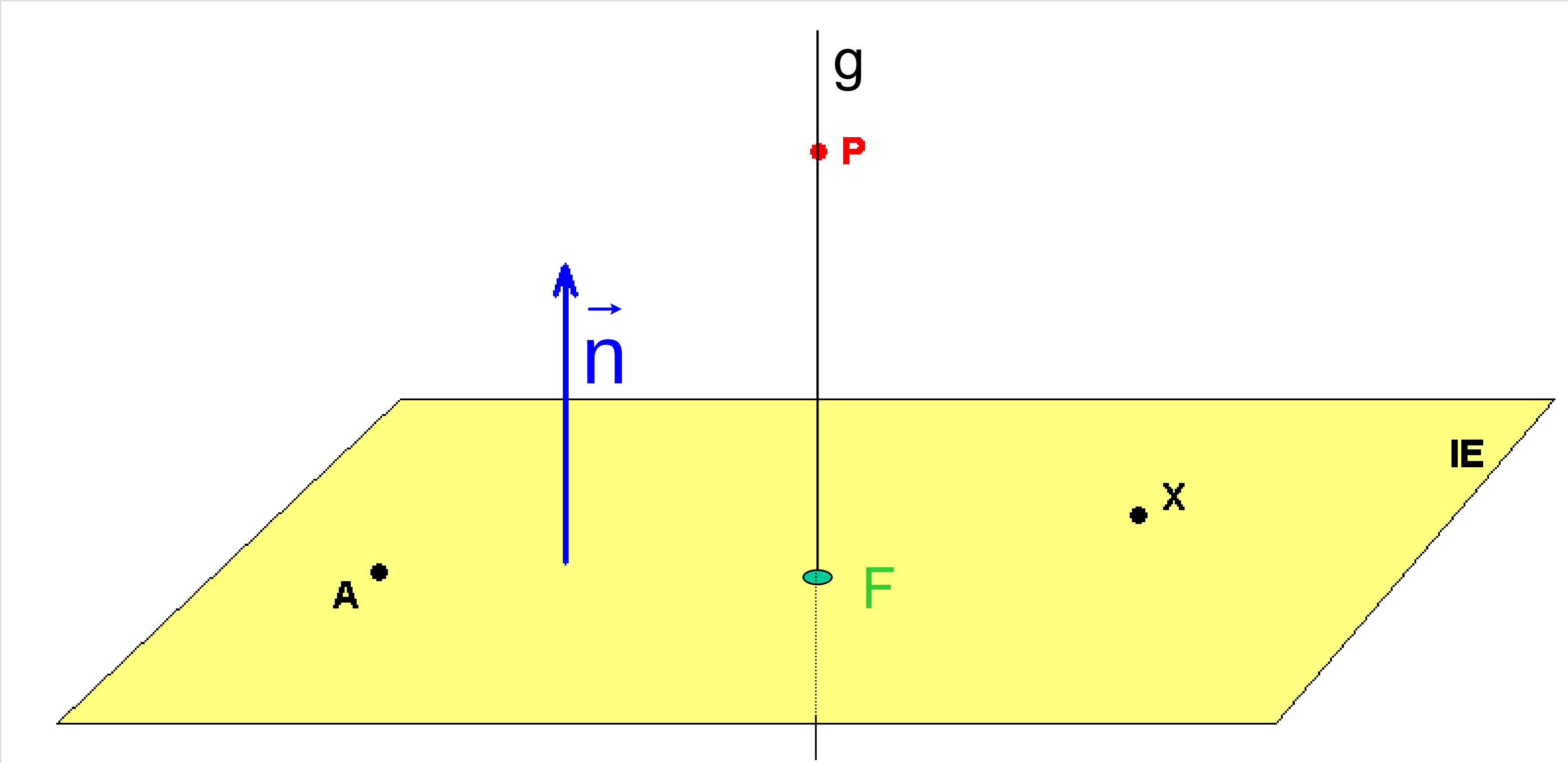
$$E_1: \quad n_{11} \cdot x_1 + n_{12} \cdot x_2 + n_{13} \cdot x_3 - c_1 = 0$$

$$E_2: \quad n_{21} \cdot x_1 + n_{22} \cdot x_2 + n_{23} \cdot x_3 - c_2 = 0$$

•k

$$\boxed{E_1 = E_2 \Leftrightarrow \overrightarrow{n_2} = k \cdot \overrightarrow{n_1} \quad \wedge c_2 = k \cdot c_1}$$

Abstand Punkt-Ebene

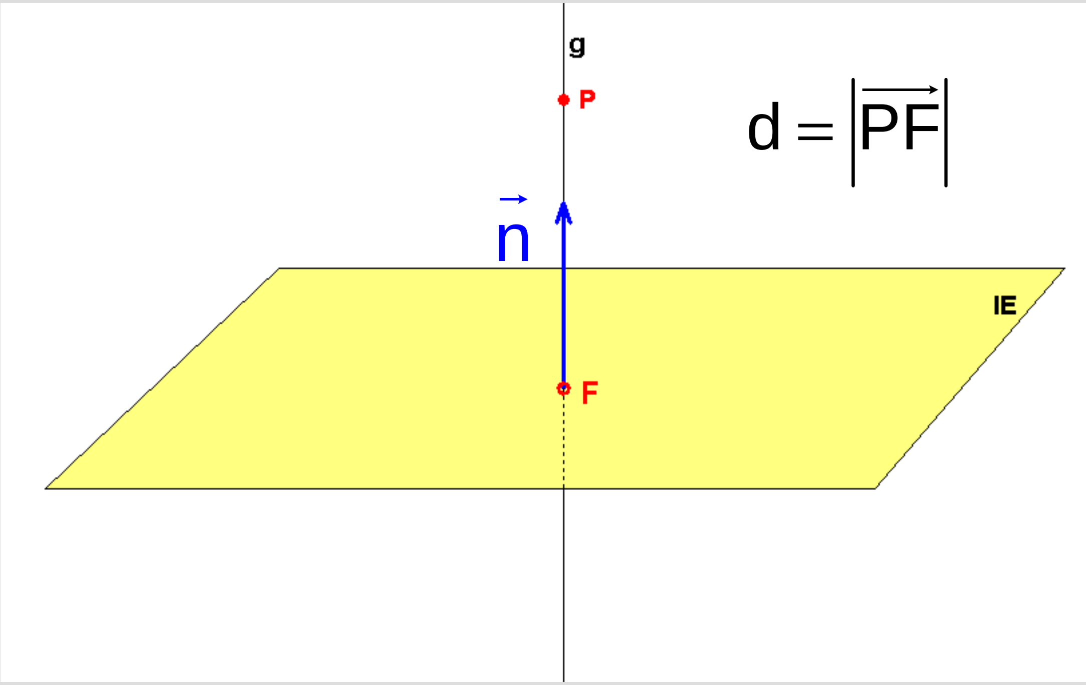


Idee: Lotgerade $g \perp E$

Lotfußpunkt F bestimmen

$$d = \left|\overrightarrow{FP}\right|$$

Abstand Punkt-Ebene Beispiel



$$d = \left|\overrightarrow{PF}\right|$$

$$E:\ \begin{pmatrix}10\\-8\\-5\end{pmatrix} * \left(\vec{x} - \begin{pmatrix}5\\2\\3\end{pmatrix}\right) = 0$$

$$P(2|-1|0)$$

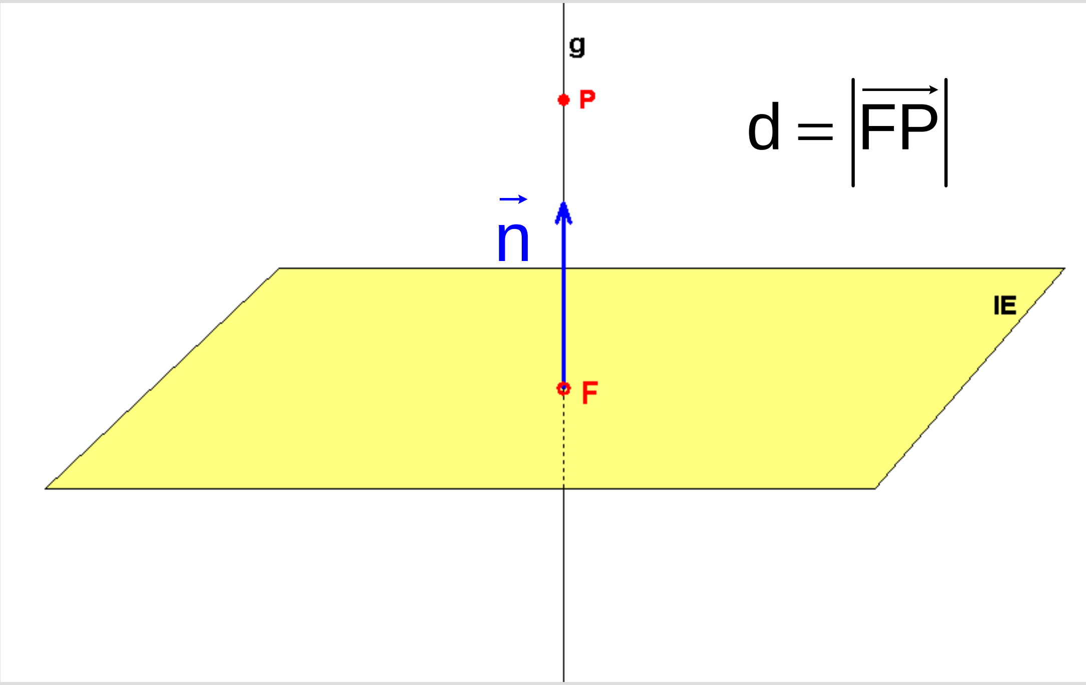
Lotgerade $g \perp E$

$$g:\ \vec{x} = \begin{pmatrix}2\\-1\\0\end{pmatrix} + t\begin{pmatrix}10\\-8\\-5\end{pmatrix} = \begin{pmatrix}2+10t\\-1-8t\\-5t\end{pmatrix}$$

Lotfußpunkt F bestimmen

$$\begin{pmatrix}10\\-8\\-5\end{pmatrix} * \left(\begin{pmatrix}2+10t\\-1-8t\\-5t\end{pmatrix} - \begin{pmatrix}5\\2\\3\end{pmatrix}\right) = 0$$

Abstand Punkt-Ebene Beispiel



$$E: \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}\right) = 0$$

$$P(2\,|\,-1\,|\,0)$$

Lotfußpunkt F bestimmen

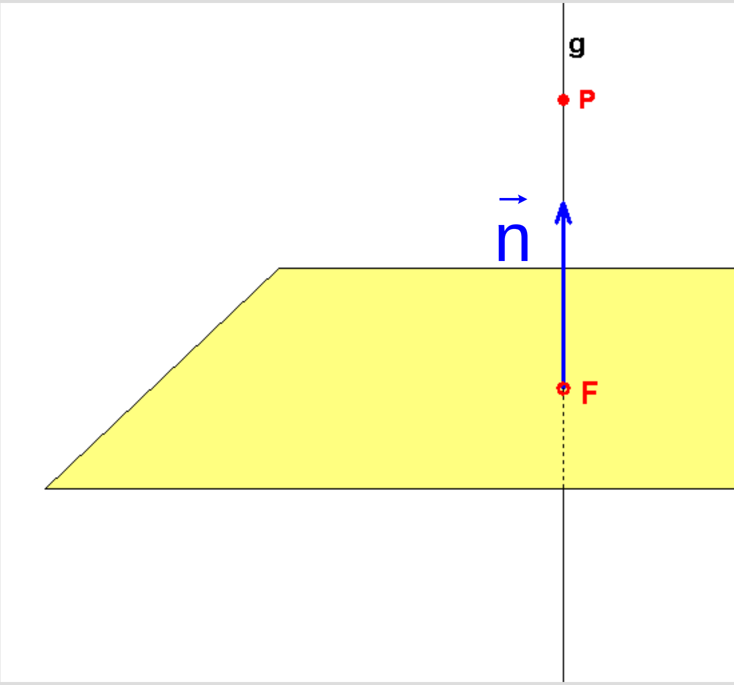
$$\begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\begin{pmatrix} -3+10t \\ -3-8t \\ -3-5t \end{pmatrix}\right) = 0$$

$$-30 + 100t + 24 + 64t + 15 + 25t = 0$$

$$-9 + 189t = 0$$

$$t = \frac{9}{189}$$

Abstand Punkt-Ebene Beispiel



$$d = \left|\overrightarrow{FP}\right|$$

$$E:\ \begin{pmatrix}10\\-8\\-5\end{pmatrix} * \left(\vec{x} - \begin{pmatrix}5\\2\\3\end{pmatrix}\right) = 0$$

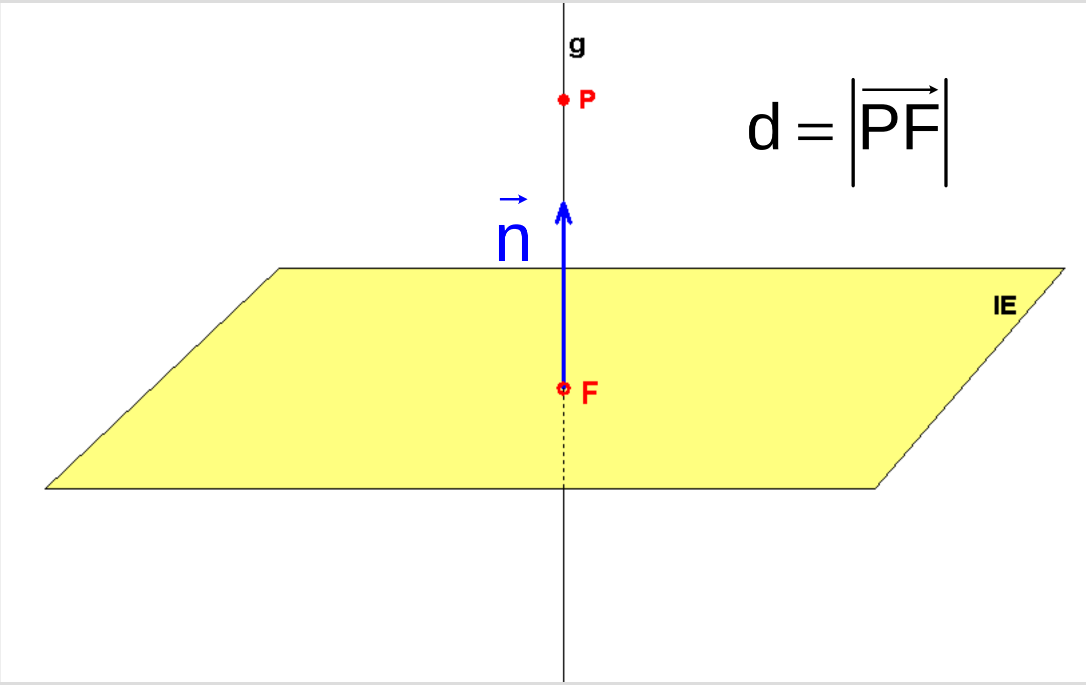
$$P(2|-1|0)$$

$$g:\ \vec{x} = \begin{pmatrix}2\\-1\\0\end{pmatrix} + t\begin{pmatrix}10\\-8\\-5\end{pmatrix}$$

Lotfußpunkt F bestimmen $t = \dfrac{9}{189}$

$$\vec{f} = \begin{pmatrix}2\\-1\\0\end{pmatrix} + \frac{9}{189}\begin{pmatrix}10\\-8\\-5\end{pmatrix} = \begin{pmatrix}2\\-1\\0\end{pmatrix} + \frac{9}{\sqrt{189}} \bullet \frac{1}{\sqrt{189}}\begin{pmatrix}10\\-8\\-5\end{pmatrix} = \begin{pmatrix}2\\-1\\0\end{pmatrix} + \frac{9}{\sqrt{189}}\overrightarrow{n_0}$$

Abstand Punkt-Ebene   Beispiel



$$d = \left|\overrightarrow{PF}\right|$$

$$E: \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}\right) = 0$$

$$P(2|-1|0)$$

$$\vec{f} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \frac{9}{\sqrt{189}} \overrightarrow{n_0}$$

$$d = \left|\overrightarrow{PF}\right| = \left|\frac{9}{\sqrt{189}} \overrightarrow{n_0}\right| = \frac{9}{\sqrt{189}}$$

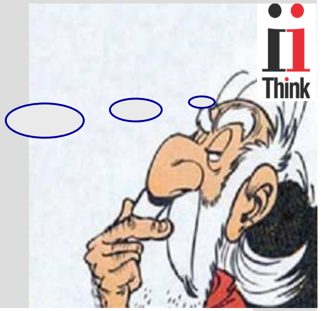
Abstand Punkt-Ebene   Beispiel

$$E: \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}\right) = 0 \qquad P(2|-1|0)$$

$$E: \quad 10 \cdot x_1 - 8 \cdot x_2 - 5 \cdot x_3 - 19 = 0$$

$$E: \quad \frac{10 \cdot x_1 - 8 \cdot x_2 - 5 \cdot x_3 - 19}{\sqrt{189}} = 0$$

Hesse, Hesse, das ist clever !



$$d = \left|\overrightarrow{PF}\right| = \left|\frac{10 \cdot 2 - 8 \cdot (-1) - 5 \cdot 0 - 19}{\sqrt{189}}\right| = \left|\frac{9}{\sqrt{189}}\right| = \frac{9}{\sqrt{189}}$$

Abstand Punkt-Ebene allgemein

$$E:\quad \vec{n} * (\vec{x} - \vec{a}) = 0 \qquad P(p_1 \,|\, p_1 \,|\, p_1)$$

$$\vec{n} * \vec{x} - \vec{n} * \vec{a} = 0$$

$$E:\quad n_1 \cdot x_1 + n_2 \cdot x_2 + n_3 \cdot x_3 - c = 0$$

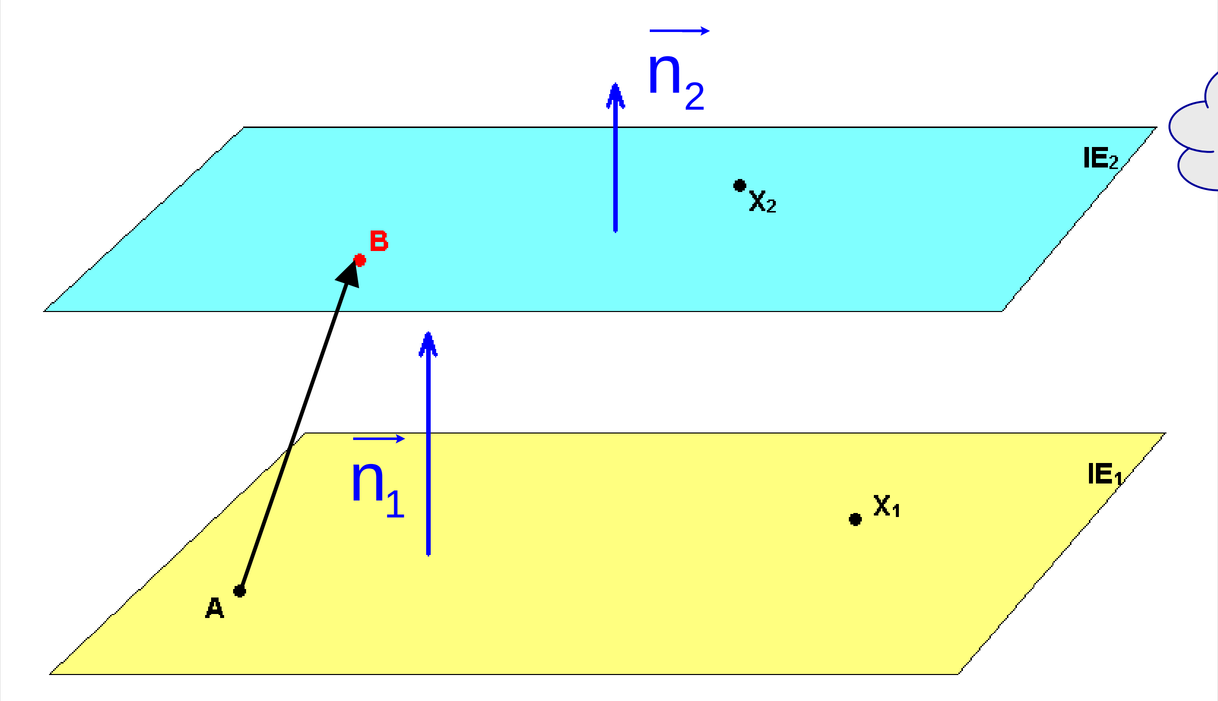
$$E:\quad \frac{n_1 \cdot x_1 + n_2 \cdot x_2 + n_3 \cdot x_3 - c}{\sqrt{n_1^2 + n_2^2 + n_3^2}} = 0$$

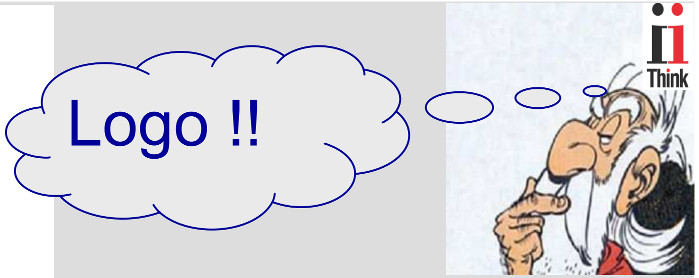
$$d = \left| \vec{n_0} * (\vec{p} - \vec{a}) \right|$$

Hesse, Hesse, das ist clever !

$$d = \left|\overrightarrow{PF}\right| = \left| \frac{n_1 \cdot p_1 + n_2 \cdot p_2 + n_3 \cdot p_3 - c}{\sqrt{189}} \right|$$

Abstand Ebene-Ebene





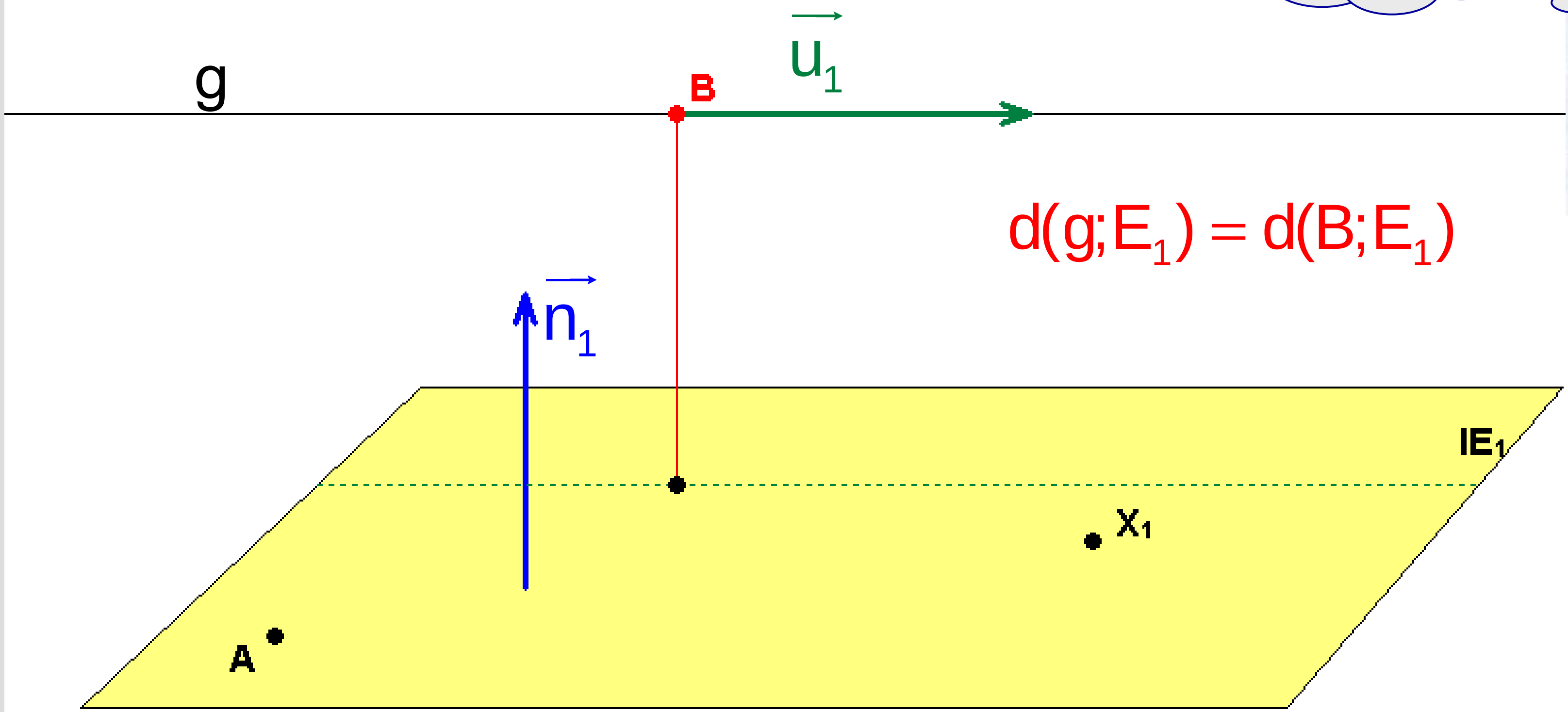
$$d(E_1;E_2) = d(B;E_1)$$

$$E_1 \,||\, E_2 \quad \Leftrightarrow \vec{n_2} = k \cdot \vec{n_1}$$

$$(E_1 \,||\, E_2 \wedge E_1 \neq E_2) \quad \Leftrightarrow (\vec{n_2} = k \cdot \vec{n_1} \wedge \vec{n_1} * (\vec{b} - \vec{a}) \neq 0)$$

$$E_1 = E_2 \quad \Leftrightarrow (\vec{n_2} = k \cdot \vec{n_1} \wedge \vec{n_1} * (\vec{b} - \vec{a}) = 0)$$

Abstand Gerade-Ebene



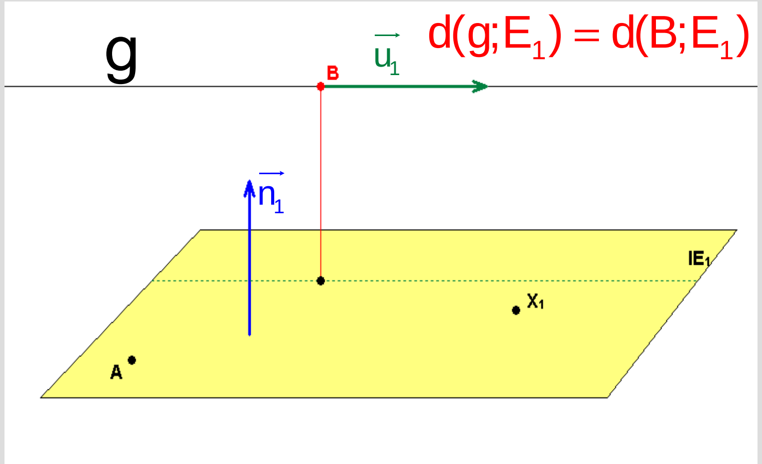
$$d(g;E_1) = d(B;E_1)$$

$$g \,||\, E_1 \quad \Leftrightarrow \overrightarrow{n_2} * \vec{u} = 0$$

$$(g \,||\, E_1 \wedge g \not\subset E_1) \quad \Leftrightarrow (\overrightarrow{n_2} * \vec{u} = 0 \wedge \overrightarrow{n_1} * (\vec{b} - \vec{a}) \neq 0)$$

$$g \subset E_1 \quad \Leftrightarrow (\overrightarrow{n_2} * \vec{u} = 0 \wedge \overrightarrow{n_1} * (\vec{b} - \vec{a}) = 0)$$

Abstand Gerade-Ebene Beispiel



$$g: \quad \vec{x} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0,4 \end{pmatrix}$$

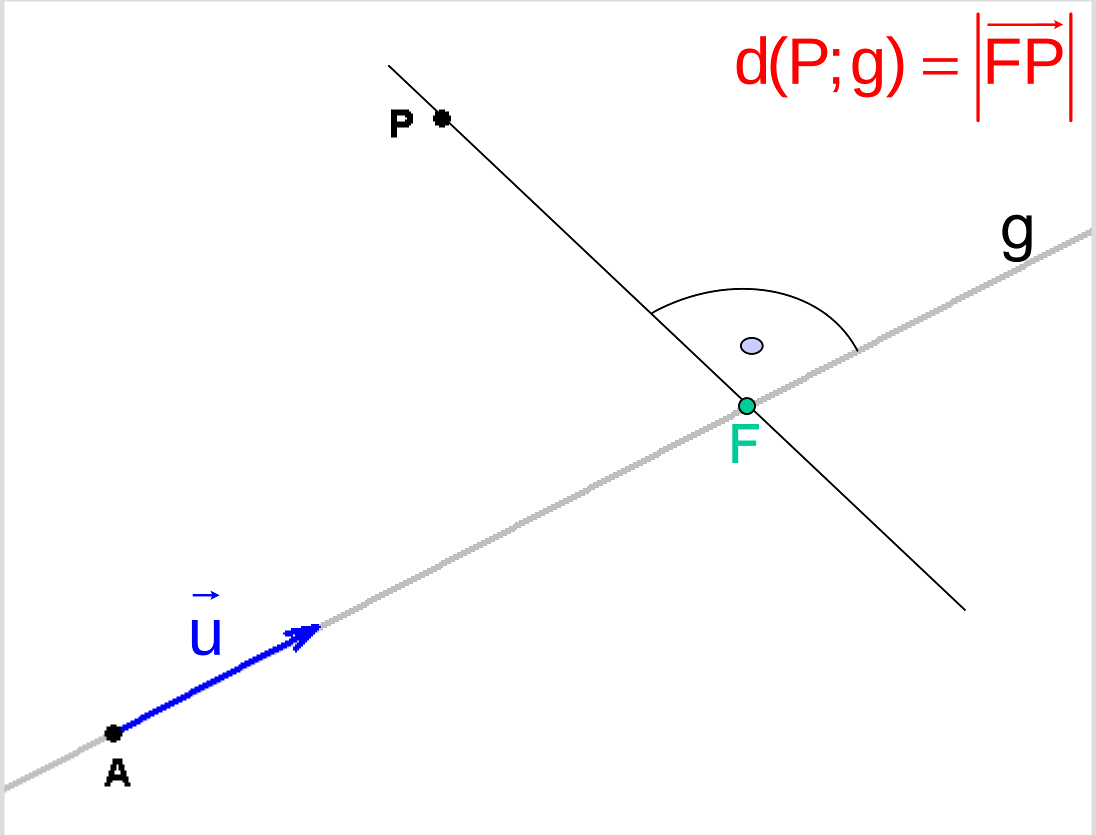
$$E: \quad \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}\right) = 0$$

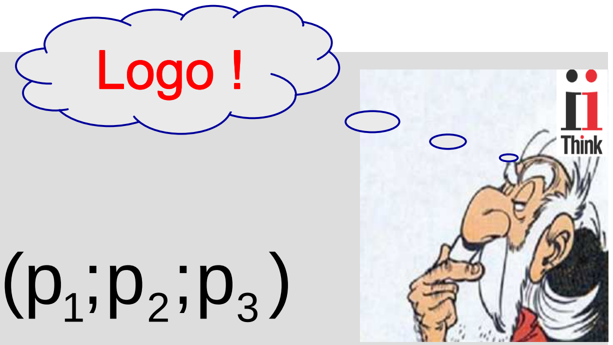
$$g \parallel E_1 \wedge g \not\subset E_1 \quad : \quad \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} * \begin{pmatrix} 1 \\ 1 \\ 0,4 \end{pmatrix} = 0 \qquad \left(\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}\right) * \begin{pmatrix} 10 \\ -8 \\ -5 \end{pmatrix} = 20 - 32 - 10 \neq 0$$

$$E: \quad \frac{10 \cdot x_1 - 8 \cdot x_2 - 5 \cdot x_3 - 19}{\sqrt{189}} = 0$$

$$d(B;E) = \left| \frac{10 \cdot 3 - 8 \cdot (-2) - 5 \cdot 1 - 19}{\sqrt{189}} \right| = \left| \frac{22}{\sqrt{189}} \right| = \frac{22}{\sqrt{189}}$$

Abstand Punkt-Gerade Beispiel





$P(p_1;p_2;p_3)$

$g:\ \vec{x} = \vec{a} + \lambda\vec{u}$

Idee:

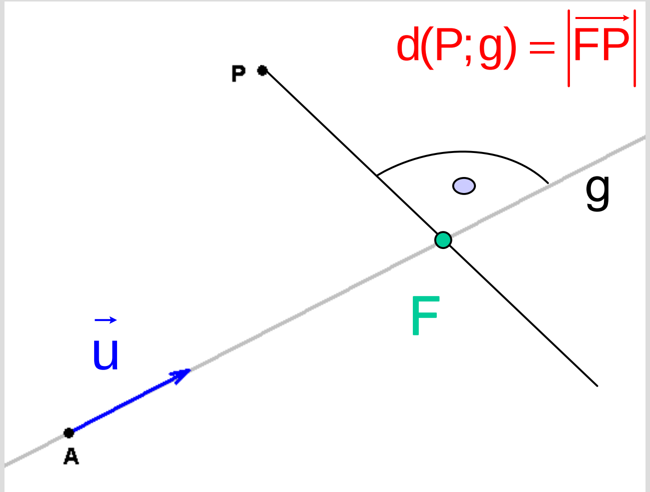
$F \in g$

$\overrightarrow{FP} \perp g$

$\vec{f} = \vec{a} + k \cdot \vec{u}$ $\qquad$ $\overrightarrow{FP} = \vec{p} - \vec{f} = \vec{p} - \vec{a} - k \cdot \vec{u}$

$\overrightarrow{FP} * \vec{u} = \vec{p} * \vec{u} - \vec{a} * \vec{u} - k \cdot \vec{u} * \vec{u} = 0 \qquad \Rightarrow k = \ldots \Rightarrow \overrightarrow{FP} = \ldots$

Abstand Punkt-Gerade Beispiel



$$g:\ \vec{x} = \begin{pmatrix}10\\18\\5\end{pmatrix} + \lambda\begin{pmatrix}2\\6\\1\end{pmatrix} \qquad P(1;2;-4)$$

Idee:

$F \in g \ \wedge \overrightarrow{FP} \perp g$

$$\vec{f} = \begin{pmatrix}10\\18\\5\end{pmatrix} + k\begin{pmatrix}2\\6\\1\end{pmatrix} \qquad \overrightarrow{FP} * \vec{u} = \left(\vec{p}-\vec{f}\right) * \vec{u} = \begin{pmatrix}1\\2\\-4\end{pmatrix} * \begin{pmatrix}2\\6\\1\end{pmatrix} - \begin{pmatrix}10\\18\\5\end{pmatrix} * \begin{pmatrix}2\\6\\1\end{pmatrix} - k \cdot 41 = 0$$

$$10 - 133 - k \cdot 41 = 0$$

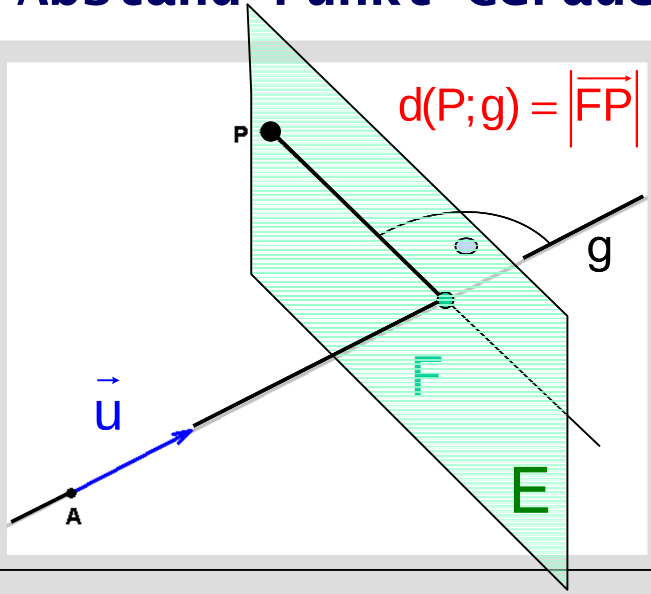
$$-123 = k \cdot 41$$

$$k = -3$$

$$\Rightarrow \vec{f} = \begin{pmatrix}4\\0\\2\end{pmatrix} \quad \Rightarrow \overrightarrow{FP} = \begin{pmatrix}-3\\2\\-6\end{pmatrix} \quad \Rightarrow \left|\overrightarrow{FP}\right| = \sqrt{49} = 7$$

$F(4\,|\,0\,|\,2)$

Abstand Punkt-Gerade 2. Möglichkeit



$$g:\ \vec{x}=\begin{pmatrix}10\\18\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\6\\1\end{pmatrix}$$

$$P(1;2;-4)$$

Idee:

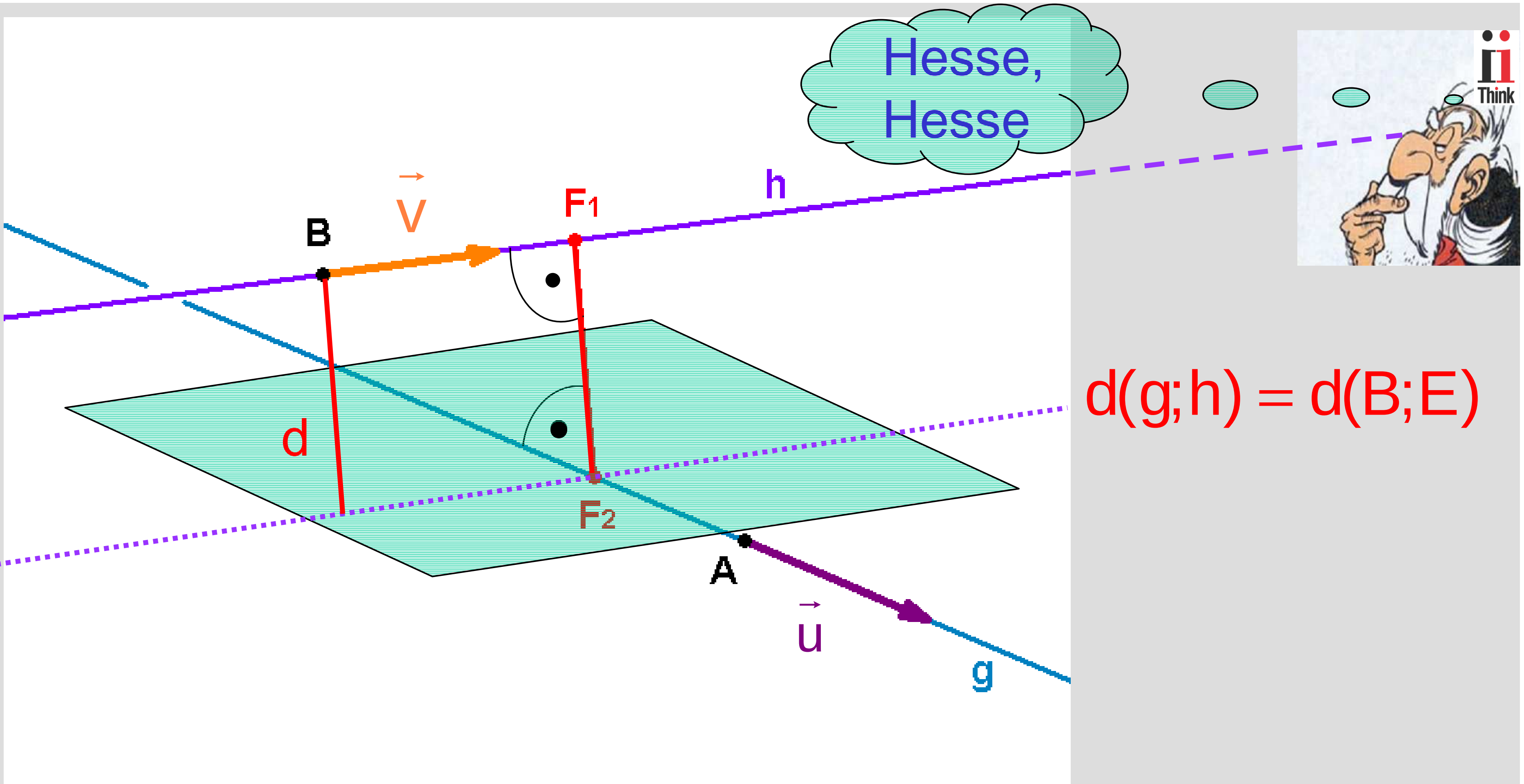
$E \perp g \wedge P \in E$

$$E:\vec{x}=\begin{pmatrix}2\\6\\1\end{pmatrix}*\left(\vec{x}-\begin{pmatrix}1\\2\\-4\end{pmatrix}\right)=0 \qquad 2x_1+6x_2+1x_3-10=0$$

$$2(10+2\lambda)+6(18+6\lambda)+1(5+\lambda)-10=0 \Rightarrow \lambda=-3$$

$$\Rightarrow \vec{f}=\begin{pmatrix}10\\18\\5\end{pmatrix}-3\begin{pmatrix}2\\6\\1\end{pmatrix}=\begin{pmatrix}4\\0\\2\end{pmatrix} \qquad \Rightarrow \overrightarrow{FP}=\begin{pmatrix}-3\\2\\-6\end{pmatrix} \qquad \Rightarrow \left|\overrightarrow{FP}\right|=\sqrt{49}=7$$

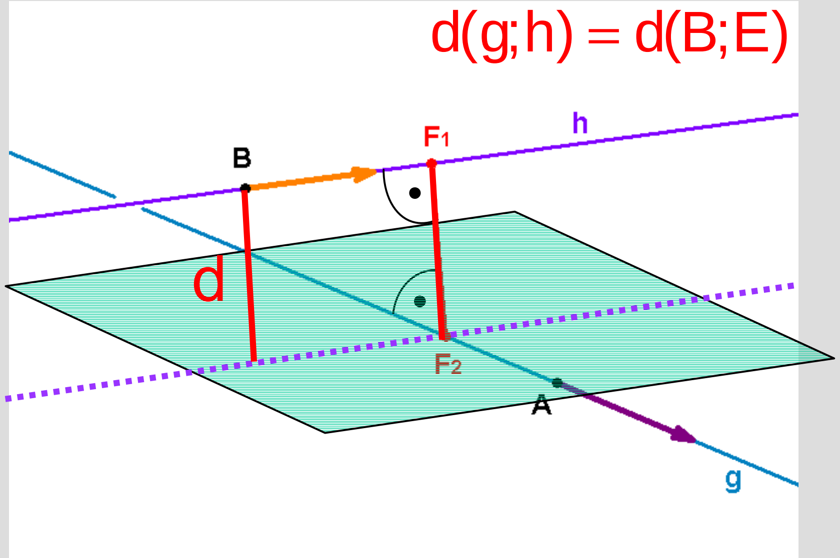
Abstand windschiefer Geraden



$d(g;h) = d(B;E)$

Lösungsidee: Ebene E, die parallel zu h verläuft und die Gerade g enthält !

Abstand windschiefer Geraden **Beispiel**



$$g: \quad \vec{x} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$h: \quad \vec{x} = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

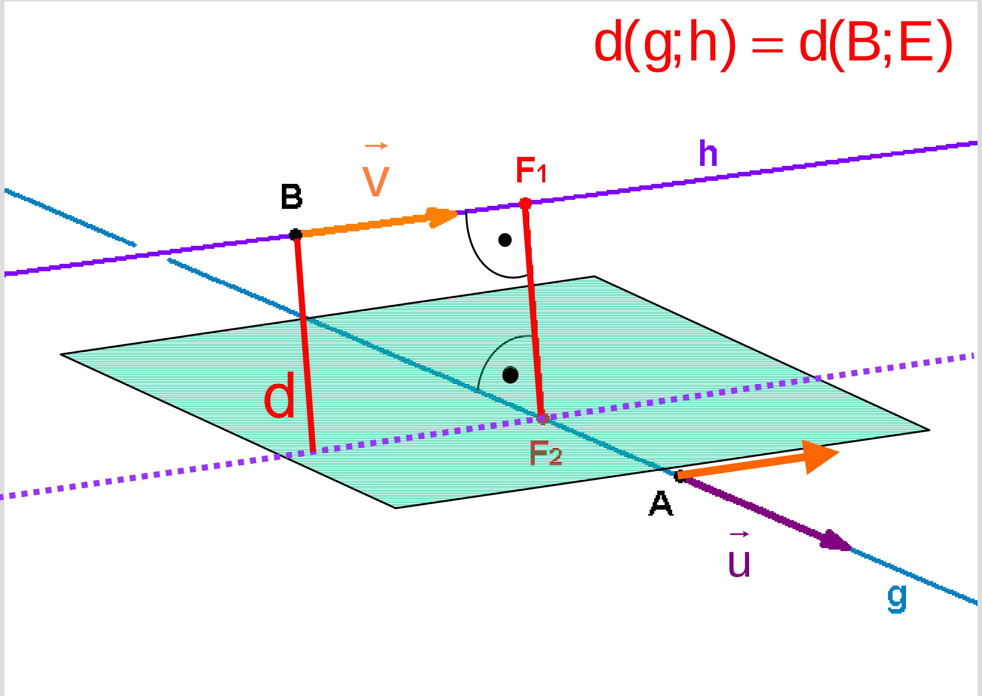
$$\vec{n} = \left| \begin{matrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 2 & 3 & 0 \\ 2 & 0 & -1 \end{matrix} \right| \begin{matrix} \vec{e}_x & \vec{e}_y \\ 2 & 3 \\ 2 & 0 \end{matrix} = \begin{pmatrix} -3 \\ 2 \\ -6 \end{pmatrix}$$

$$E: \begin{pmatrix} -3 \\ 2 \\ -6 \end{pmatrix} * \left(\vec{x} - \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix}\right) = 0$$

$$E: \frac{3 \cdot x_1 - 2 \cdot x_2 + 6 \cdot x_3 + 44}{7} = 0$$

$$d(B;E) = \left| \frac{3 \cdot 2 - 2 \cdot (-3) + 6 \cdot 7 + 44}{7} \right| = \left| \frac{98}{7} \right| = 14$$

Abstand windschiefer Geraden Eine einfache Formel



$$g: \vec{x} = \vec{a} + \lambda\vec{u}$$

$$h: \vec{x} = \vec{b} + \lambda\vec{v}$$

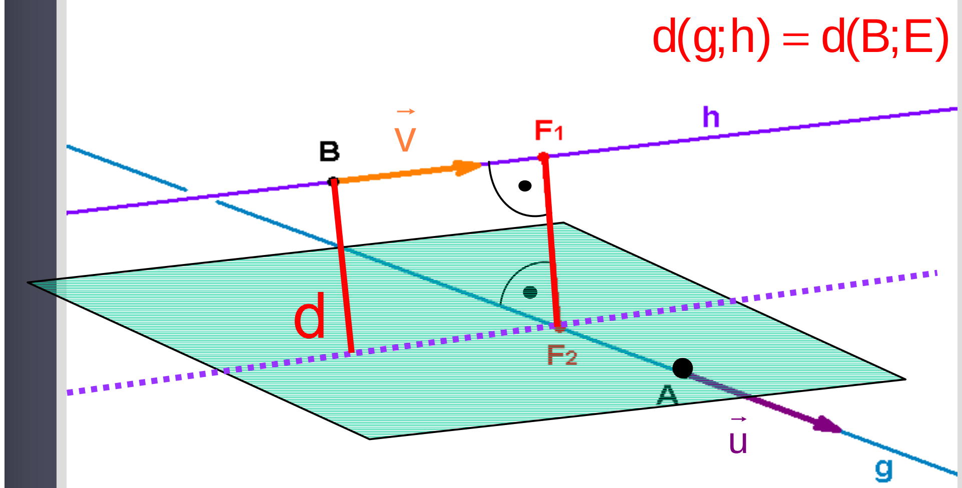
$$\vec{n} = \vec{u} \times \vec{v}$$

$$\vec{n_0} = \frac{\vec{u} \times \vec{v}}{\left|\vec{u} \times \vec{v}\right|}$$

HNF $\quad E: \frac{(\vec{u} \times \vec{v})}{\left|\vec{u} \times \vec{v}\right|} * (\vec{x} - \vec{a}) = 0$

$$d(B;E) = \left| \frac{(\vec{u} \times \vec{v})}{\left|\vec{u} \times \vec{v}\right|} * (\vec{b} - \vec{a}) \right|$$

Abstand windschiefer Geraden - Lotfußpunkte



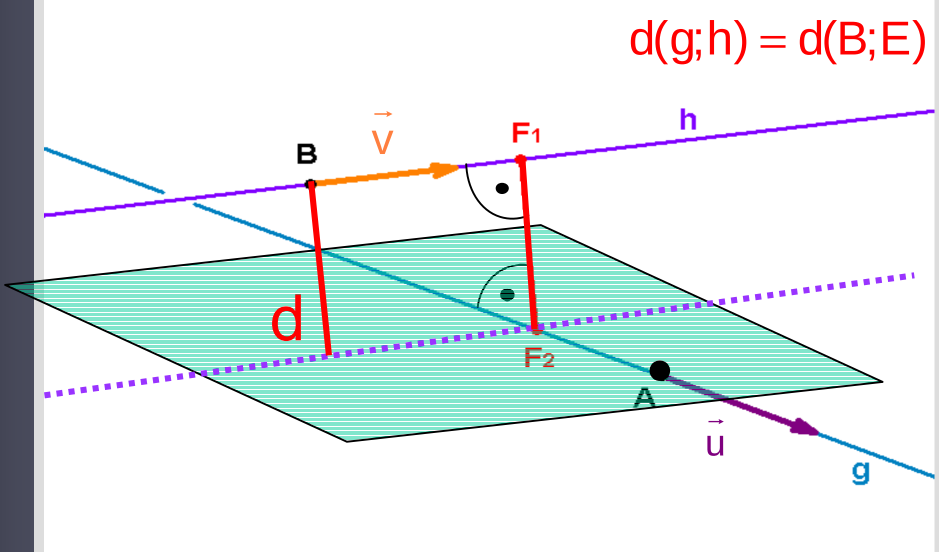
$$g: \quad \vec{x} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$h: \quad \vec{x} = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$F_1 \in h: \ \vec{f_1} = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \qquad F_2 \in g: \quad \vec{f_2} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$\vec{d} = \overrightarrow{F_1F_2} = \vec{f_2} - \vec{f_1} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} - \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 + 2\lambda - 2\mu \\ 10 + 3\lambda + 0\mu \\ -13 + 0\lambda + 1\mu \end{pmatrix}$$

Abstand windschiefer Geraden - Lotfußpunkte



$$g: \quad \vec{x} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$h: \quad \vec{x} = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$\vec{d} = \overrightarrow{F_2F_1} = \vec{f_2} - \vec{f_1} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} - \lambda \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 + 2\lambda - 2\mu \\ 10 + 3\lambda + 0\mu \\ -13 + 0\lambda + 1\mu \end{pmatrix}$$

Aus $\vec{d} * \vec{u} = 0$ und $\vec{d} * \vec{v} = 0$ folgt :

$$\begin{pmatrix} 0 + 2\lambda - 2\mu \\ 10 + 3\lambda + 0\mu \\ -13 + 0\lambda + 1\mu \end{pmatrix} * \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = 0 \qquad \begin{pmatrix} 0 + 2\lambda - 2\mu \\ 10 + 3\lambda + 0\mu \\ -13 + 0\lambda + 1\mu \end{pmatrix} * \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = 0$$

Abstand windschiefer Geraden - Lotfußpunkte

Aus $\vec{d} * \vec{u} = 0$ und $\vec{d} * \vec{v} = 0$ folgt :

$$\begin{pmatrix} 0 + 2\lambda - 2\mu \\ 10 + 3\lambda + 0\mu \\ -13 + 0\lambda + 1\mu \end{pmatrix} * \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = 0 \qquad \begin{pmatrix} 0 + 2\lambda - 2\mu \\ 10 + 3\lambda + 0\mu \\ -13 + 0\lambda + 1\mu \end{pmatrix} * \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = 0$$

$$\left| \begin{array}{l} 4\lambda - 4\mu + 30 + 9\lambda = 0 \\ 4\lambda - 4\mu + 13 - \mu = 0 \end{array} \right|$$

$$\left| \begin{array}{l} 13\lambda - 4\mu = -30 \\ 4\lambda \quad - 5\mu = -13 \end{array} \right|$$

$$5 \cdot \mathrm{I} - 4 \cdot \mathrm{II} \ : \ 49\lambda = -98 \Rightarrow \lambda = -2$$

$$\mu \text{ aus II} \ : \ 4\lambda + 13 = 5\mu$$

$$5 = 5\mu \Rightarrow \mu = 1$$

$$\vec{f_2} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} - 2 \cdot \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -6 \end{pmatrix}$$

$$\vec{f_1} = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} + 1 \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix}$$

Abstand windschiefer Geraden - Lotfußpunkte

$$\vec{f_2} = \begin{pmatrix} 2 \\ 7 \\ -6 \end{pmatrix} - 2 \cdot \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -6 \end{pmatrix}$$

$$\vec{f_1} = \begin{pmatrix} 2 \\ -3 \\ 7 \end{pmatrix} + 1 \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix}$$

$$\vec{d} = \vec{f_1} - \vec{f_2} = \begin{pmatrix} -6 \\ 4 \\ 12 \end{pmatrix}$$

$$\left|\vec{d}\right| = \sqrt{36+16+144} = \sqrt{196} = 14$$

Sic itur ad astra !

So steigt man zu den Sternen empor !